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C ABAQUS User subroutine UMAT
C The subroutine implements graded elements with variable Young's
C modulus and Poisson's ration in accordance with Mori-Tanaka
C scheme into ABAQUS/Standard.
C The subroutine handles for 3-D problems.
C
C If using this code for research or industrial purposes, please
C cite: Vycheslav N. Burlayenko & Tomasz Sadowski, Free vibrations
C and static analysis of functionally graded sandwich plates with
C three-dimensional finite elements, Meccanica (2019)
C
C NOTE: FGM PLATE with material gradation through-the-
C thickness. HERE it means through the Y-axis
C
      SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
     1 RPL,DDSDDT,DRPLDE,DRPLDT,STRAN,DSTRAN,
     2 TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,MATERL,NDI,NSHR,NTENS,
     3 NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,CELENT,
     4 DFGRD0,DFGRD1,NOEL,NPT,KSLAY,KSPT,KSTEP,KINC)
C
      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 MATERL
      DIMENSION STRESS(NTENS),STATEV(NSTATV),
     1 DDSDDE(NTENS,NTENS),DDSDDT(NTENS),DRPLDE(NTENS),
     2 STRAN(NTENS),DSTRAN(NTENS),TIME(2),PREDEF(1),DPRED(1),
     3 PROPS(NPROPS),COORDS(3),DROT(3,3),
     4 DFGRD0(3,3),DFGRD1(3,3)
C
C     LOCAL ARRAYS
C
      DIMENSION EELAS(6),ETHERM(6),DTHERM(6)
C
      PARAMETER (ZERO=0.D0,ONE=1.D0,TWO=2.D0,TWOTHIRD=2.D0/3.D0,
     1 HALF=1.D0/2.D0, THREE=3.0D0,FOUR=4.0D0,FOURTHIRD=4.D0/3.D0)

      REAL*8 E,NU,MU,LAMBDA,PHAS,AKM,AKC,AGM,AGC,FUM,AKE,AGE
C
      DATA THICKNESS, VOLC /50.0D0,1.0D0/
C
C     DETERMINNE MATERAILS PROPERTIES BASED ON GLOBAL COORDINATES
IN GAUSS POINTS.
C
C     COORDS(1)  is  X-coordinate of  Gauss points.
C     COORDS(2)  is  Y-coordinate of  Gauss points.
C     COORDS(3)  is  Z-coordinate of  Gauss points.
C
C     USER SPECIFY PARAMETERS FOR THE SUBROUTINE
C
C     PROPS(1) IS Em : Young's modulus of metal on bottom surface
C     PROPS(2) IS Ec : Young's modulus of ceramic on top surface
C     PROPS(3) IS p  : the exponent
C     PROPS(4) IS NUm : Poisson's coefficient - constant
C     PROPS(5) IS NUc : Poisson's coefficient - constant
C
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C     CHANGE FOR NEW PROBLEM
C     PHAS = ZERO + VOLC*(HALF + COORDS(2)/THICKNESS)**PROPS(3)
C
C     BULK MODULUS
      AKM = PROPS(1)/THREE/(ONE-TWO*PROPS(4))
      AKC = PROPS(2)/THREE/(ONE-TWO*PROPS(5))
C     SHEAR MODULUS
      AGM = PROPS(1)/TWO/(ONE+PROPS(4))
      AGC = PROPS(2)/TWO/(ONE+PROPS(5))
C    FUM=AGM*(THREE*THREE*AKM+FOUR*TWO*AGM)/THREE/TWO/(AKM+TWO*AGM)
C
C     EFFECTIVE MATERIAL PARAMETERS
     AKE = AKM + (AKC-AKM)*PHAS/(1 + (1-PHAS)*(AKC-AKM)/(AKM +
     1 FOURTHIRD*AGM))
     AGE = AGM + (AGC-AGM)*PHAS/(1 + (1-PHAS)*(AGC-AGM)/(AGM +
     1 FUM))
C
      E = THREE*THREE*AKE*AGE/(THREE*AKE+AGE)
      NU = (THREE*AKE-TWO*AGE)/TWO/(THREE*AKE+AGE)
C
C     DETERMINE THE LAME's CONSTANTS
C
      MU=E/TWO/(ONE + NU)
      LAMBDA=E*NU/(ONE+NU)/(ONE-TWO*NU)
C
C     DETERMINE THE TANGENT(JACOBIAN) MATRIX
C
C     NDI  IS  NUMBER OF NORMAL STRESSES:  SXX, SYY, SZZ
C     NSHR IS  NUMBER OF SHEAR STRESSES:  SXY
C     NTENS  IS  DIMENSION OF CONSTITUTIVE MATRIX,NTENS=NDI + NSHR
C
      DO I=1,NDI
        DO J=1,NDI
           IF (I.EQ.J) THEN
                DDSDDE(I, I)= LAMBDA + TWO*MU
           ELSE
                DDSDDE(I, J)= LAMBDA
           ENDIF
        ENDDO
      ENDDO
C
      DO I=NDI+1,NTENS
              DDSDDE(I, I)= MU
      ENDDO
C
C     DETERMINE THE STRESS AND UPDATE THE STRESS
C
      DO I=1,NTENS
        DO J=1,NTENS
             STRESS(I)=STRESS(I) + DDSDDE(I, J)*DSTRAN(J)
        ENDDO
      ENDDO
C
      RETURN
      END
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