INTERFACIAL FRACTURE IN SANDWICH COMPOSITES. PART 1: Basics of Fracture Mechanics

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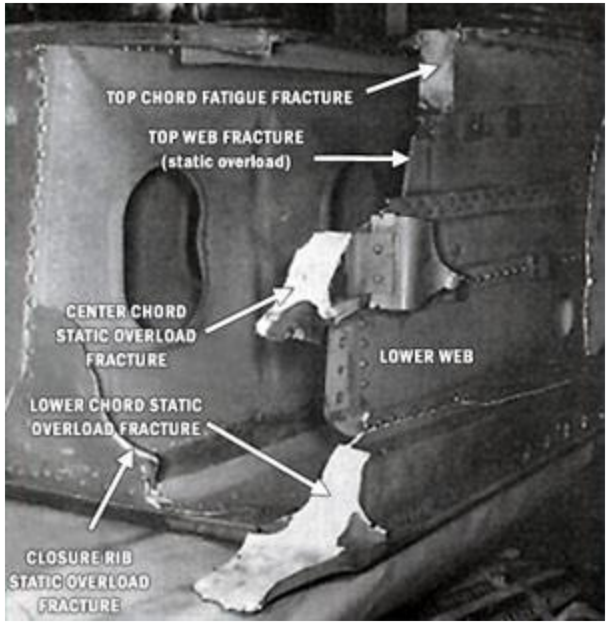
Fracture Phenomena in Engineering Applications

Silver Bridge collapse at Ohio in 1967, USA



Horizontal Stabilizer fracture in Dan-Air aircraft 1977

The S.S. Schenectady split apart by brittle fracture while in harbor, 1943.

Strength and Fracture Behaviours in Engineering

Many different areas in engineering are concerned with evaluating the fracture strength and durability of constructions. An introductory classification will be given in the following:

- **Theory of strength** acts on the assumption of deformable bodies of given geometry (G), which are free of any defects and pose an ideal continuum. The strains and stresses inside the component are determined as a result of the external load (L) by assuming a specific deformation law (elasticity, plasticity, etc.) of the material (M). Based on these results strength parameters are calculated mostly in terms of effective stress σ_v, which characterizes the stress state in each material point. Using tests on simple samples with elementary loading conditions (e. g. tension test), critical values σc of the material's strength are determined. To guarantee the safety of components, the maximum occurring loads need to stay below the critical strength parameters:

$$\sigma_v(G, L, M) \leq \sigma_{tol}(M) = \frac{\sigma_c}{S}.$$

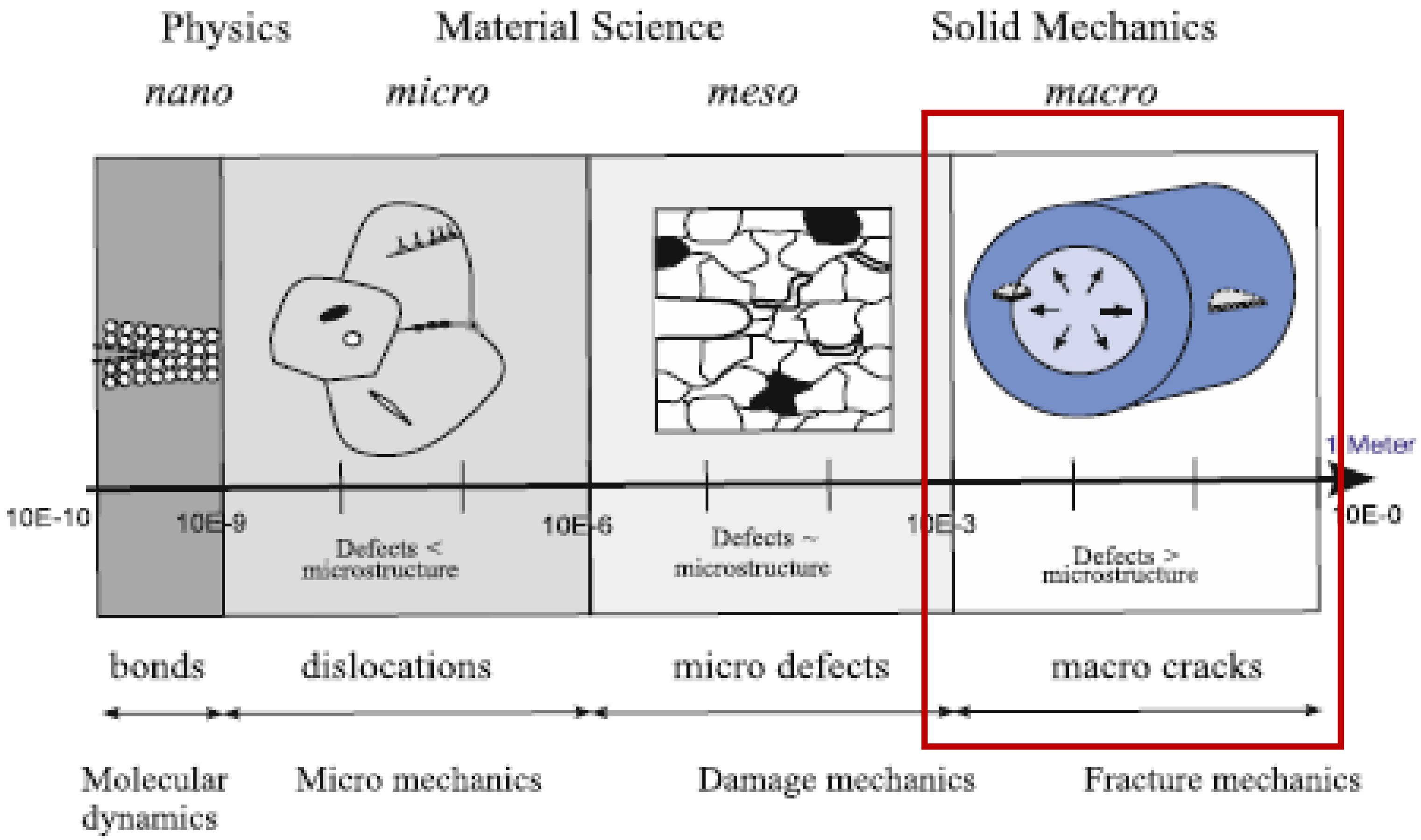
- However, the traditional strength theories and the therein used material parameters (yield strength, tensile strength, endurance limit, ultimate strain, impact energy) often fail in predicting fracture processes. The reason for this is that fracture processes primarily originate from points of concentrated stresses on crack-like defects. In such cases, the classical strength criteria provide no quantitative correlation between loading situation, geometry and material property. **Continuum damage mechanics** is the appropriate approach. The same methods as in classical strength theory are used with the difference that when expressing the law of materials, it is assumed that the material possesses small continuously distributed defects, e. g. microcracks or micropores measured by the damage D. It may change in the course of stress until a critical threshold D_c of the damage is reached. Hence, a **local failure criterion** in the form of:

$$D(G, L, M) \leq D_c(M)$$

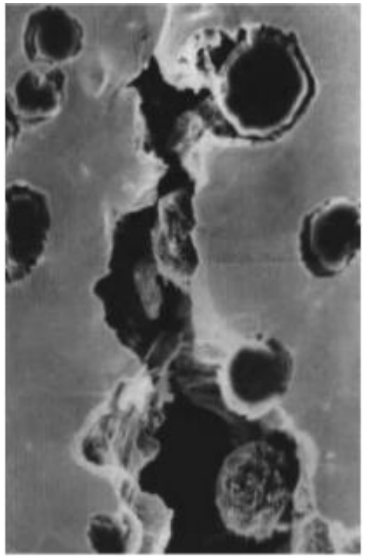
- The field which deals with fracture and failure processes in engineering materials and constructions is called **Fracture mechanics**. In contrast to the two abovementioned theories, in fracture mechanics it is assumed that every component and every real material inevitably possesses flaws or other defects modelled as ***cracks*** of the size ***a***. Analogously to the above-mentioned theories, a **fracture mechanical strength criterion** has the form:

$$B(G, L, M, a) \leq B_c(M)$$

Scientific Concepts for Assessment of Fracture



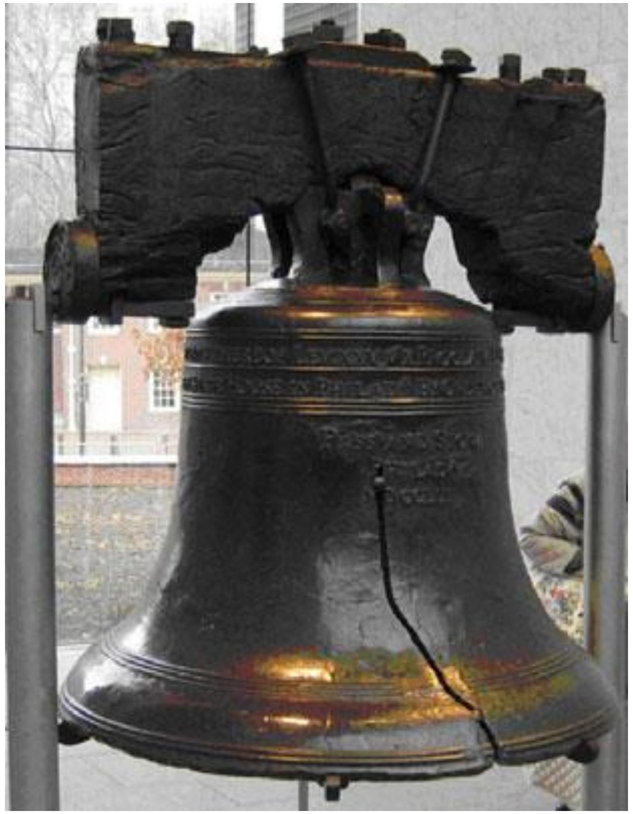
Micro-crack



Macro-crack

Macroscopic Manifestations of Fracture

Deformation-poor, or macroscopically brittle fracture

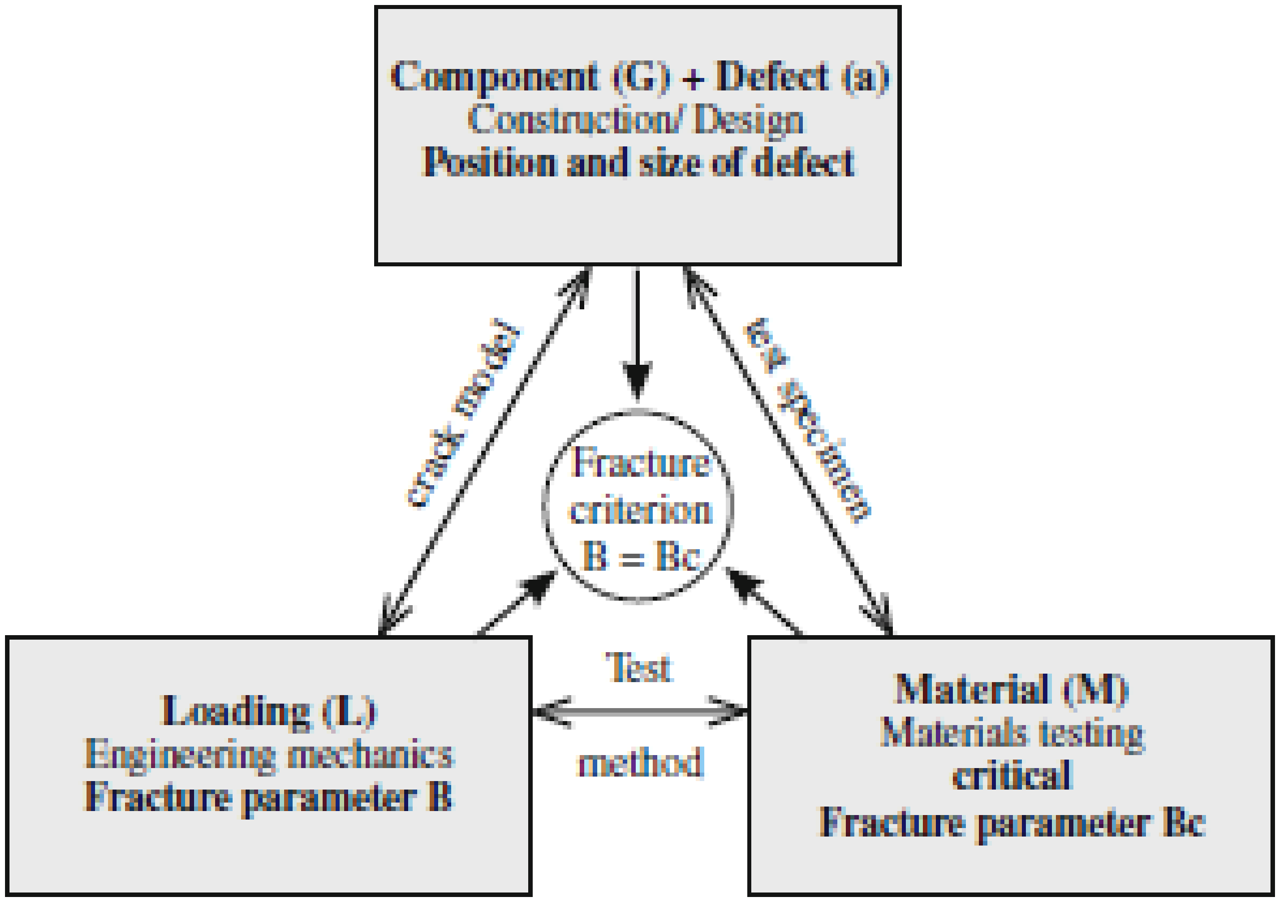


Brittle fracture of the
Liberty Bell, Philadelphia 1752

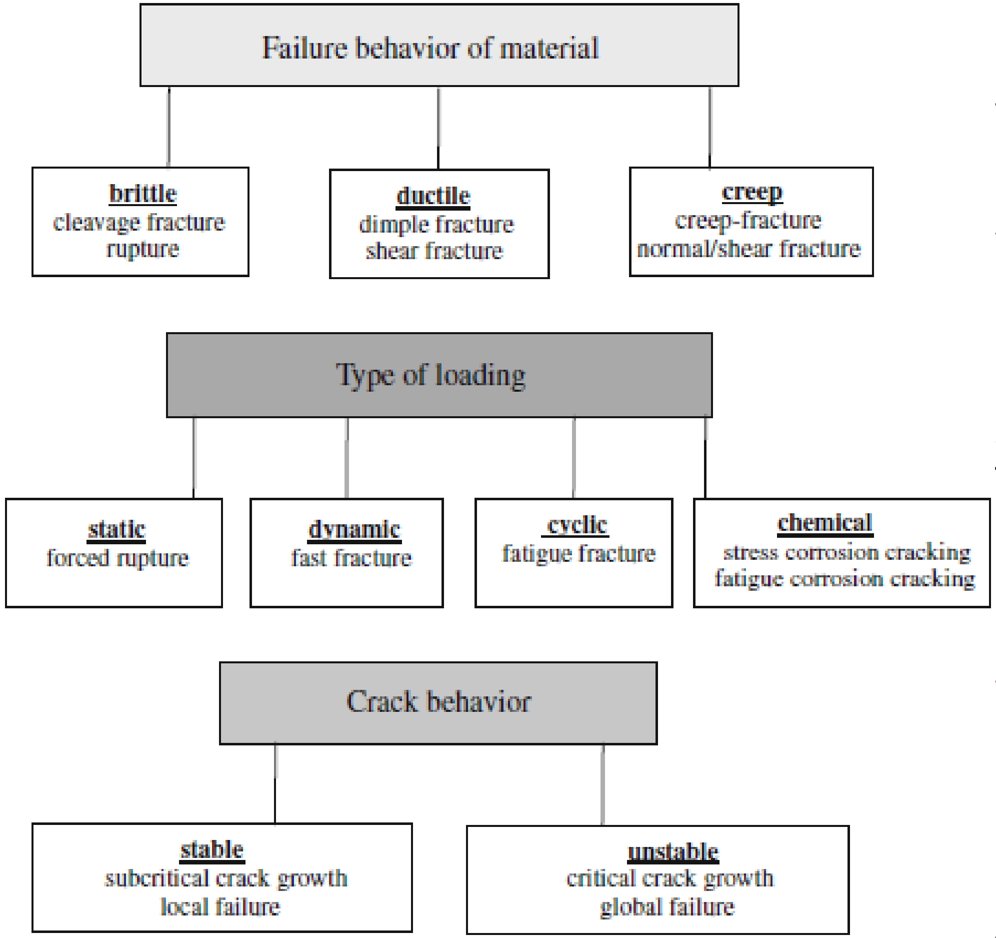
Deformation-rich, or macroscopically ductile fracture appears

Ductile failure of a specimen strained axially

Fracture-Mechanical Assessment Concept



General Classification of Fracture Processes



In the initial situation, a crack has a specific size and shape. As long as it does not change, the crack is regarded as a static or stationary crack. The moment in which the crack propagation starts due to critical loading, is called crack initiation. The crack size now increases and the crack is called unsteady.

An important feature of fracture is the stability of the crack propagation. The fracture process is then marked as unstable if the crack grows abruptly without the need to increase external loading. In contrast to this, if an additional increase of the external load is necessary in order to let the crack grow further, it is called stable crack growth.

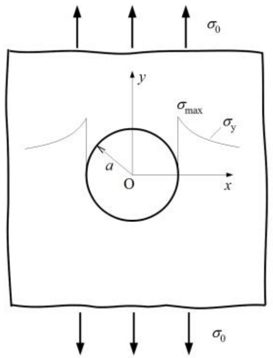
There are fracture processes that happen far below the critical load and develop in a stable manner with a very low rate of growth. To describe them, the term subcritical crack growth is introduced.

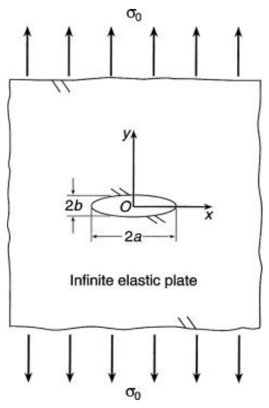
In contrast to the dynamic, impulsive load of a stationary crack, the dynamics of the fracture process itself will be considered. In most cases the crack propagation happens so slowly, that all dynamic effects in the structure may be neglected. In that case a quasi-static analysis is sufficient. If the crack growth rate reaches the level of acoustic wave speeds in the solid, velocity terms, inertia forces as well as interactions between the crack and the sound waves need to be taken into account. Additionally, to that failure mechanisms in the material depend on the deformation rate, which mostly leads to an embrittlement on fast running cracks. In this way, dynamic crack growth processes have already caused catastrophic failures

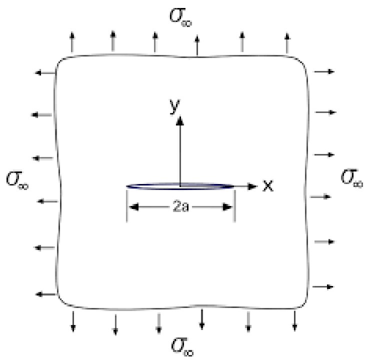
Milestones of Fracture Mechanics

1. Stress Concentrations at Holes, Kirsch, 1898
2. Wieghardt’s asymptotic singular solution, 1907
3. Stresses at Elliptical Holes, Inglis, 1913
4. Griffith's Energy Release Rate, 1920
5. Westergaard's Solution for Cracks, 1937
6. William’s asymptotic stress and displacement fields at the crack tip, 1952
7. Stress Intensity Factor, Irwin, 1957
8. J-integral, Cherepanov, 1967 and Rice, 1968
9. Dugdale’s plastic zone models, 1960
10. Well’s elasto-plastic fracture criterion, 1961

linear elastic solution for stresses and displacements around a hole in an infinite plate



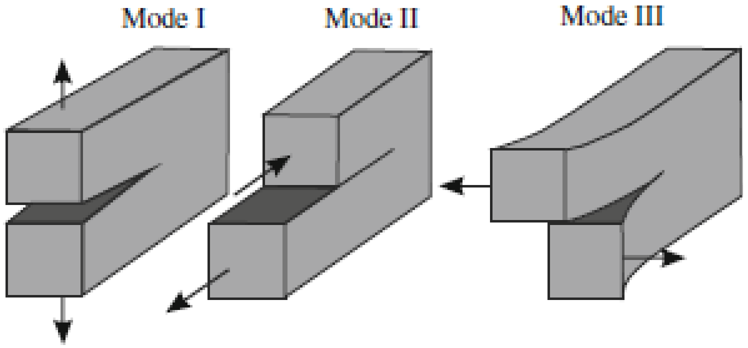




Linear Fracture Mechanics Assumptions

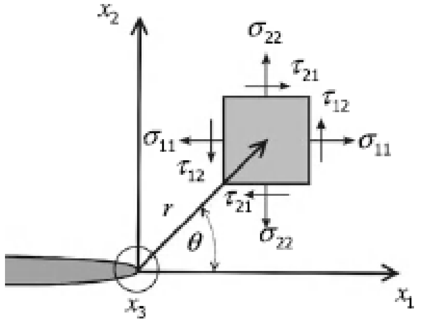
In linear-elastic fracture mechanics, crack problems are analyzed in bodies whose deformation behavior can be assumed to be linear-elastic according to the generalized Hooke's law

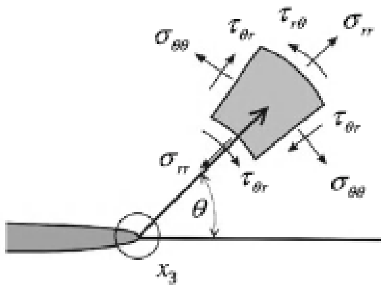
I. Definition of the crack opening modes: Every type of crack deformation can be regarded as a superposition of these three basic kinematic modes. In the three-dimensional case, is to be understood as a local section around a segment of the crack front, whereas the size of the modes will change along the crack front



II. Stresses at the crack tip in Cartesian and polar co-ordinates - Williams solution(no load is specified)

The strength of this crack-tip field is entirely determined by the Stress Intensity Factors





$$\sigma_{ij}(r,\theta) = \sum_{n=1}^{\infty} r^{\frac{n}{2}-1}\left[a_n \tilde{M}_{ij}^{(n)}(\theta) + b_n \tilde{N}_{ij}^{(n)}(\theta) + c_n \tilde{L}_{ij}^{(n)}(\theta)\right]$$

$$u_i(r,\theta) = \frac{1}{2\mu}\sum_{n=1}^{\infty}\left[a_n \tilde{F}_{i}^{(n)}(\theta) + b_n \tilde{G}_{i}^{(n)}(\theta) + c_n \tilde{H}_{i}^{(n)}(\theta)\right].$$

asymptotic crack-tip fields

$$\begin{Bmatrix} K_{\mathrm{I}} \\ K_{\mathrm{II}} \\ K_{\mathrm{III}} \end{Bmatrix} = \lim_{r\to 0}\sqrt{2\pi r}\begin{Bmatrix} \sigma_{22}(r,\theta=0) \\ \tau_{21}(r,\theta=0) \\ \tau_{23}(r,\theta=0) \end{Bmatrix}$$

The Concept of Stress Intensity Factors

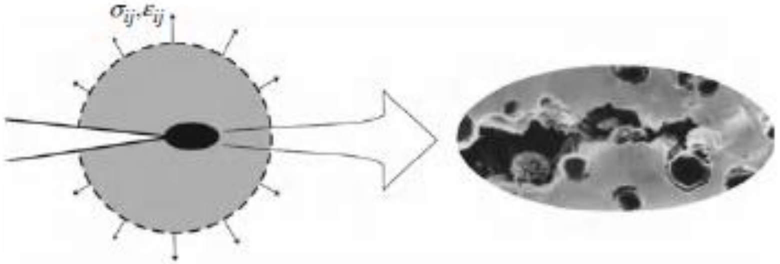
$$K_{\mathrm{I}} = K_{\mathrm{I}}\,(\text{geometry, fracture, load, material})$$

- The singular crack tip solution with coefficient K_I describes the loading state in a finite region around the crack tip with radius r_K . At a greater distance $r > r_K$, its dominance fades because other terms of the series gain influence
- It is assumed that the size r_B of the process zone is much smaller than the domain r_K of validity of K_I–solution, all fracture processes are controlled by the near field solution, acting as «boundary condition» . Given the same K_I, the same process happens, independently of which type of crack configuration is present

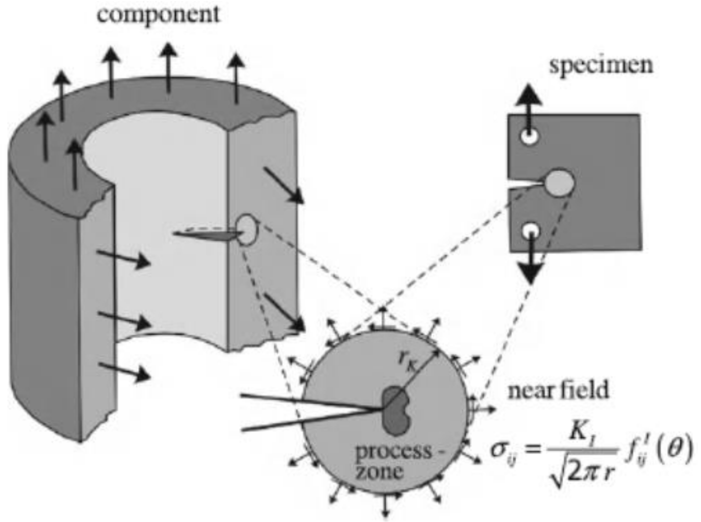
- By means of this «autonomy principle of crack tip singularity» we have reduced the entire body geometry and loading to the K_I–factor
- The crack propagation will initiate just when a critical material state in the process zone is reached. This material-specific limit value of load carrying capacity is called **fracture toughness**. The stress intensity concept provides the fracture criterion as follows: $K_{\mathrm{I}} = K_{\mathrm{Ic}}$
- or in the general case of combined loading:

$$B(K_{\mathrm{I}},\ K_{\mathrm{II}},\ K_{\mathrm{III}}) = B_{\mathrm{c}}$$

K-dominance zone

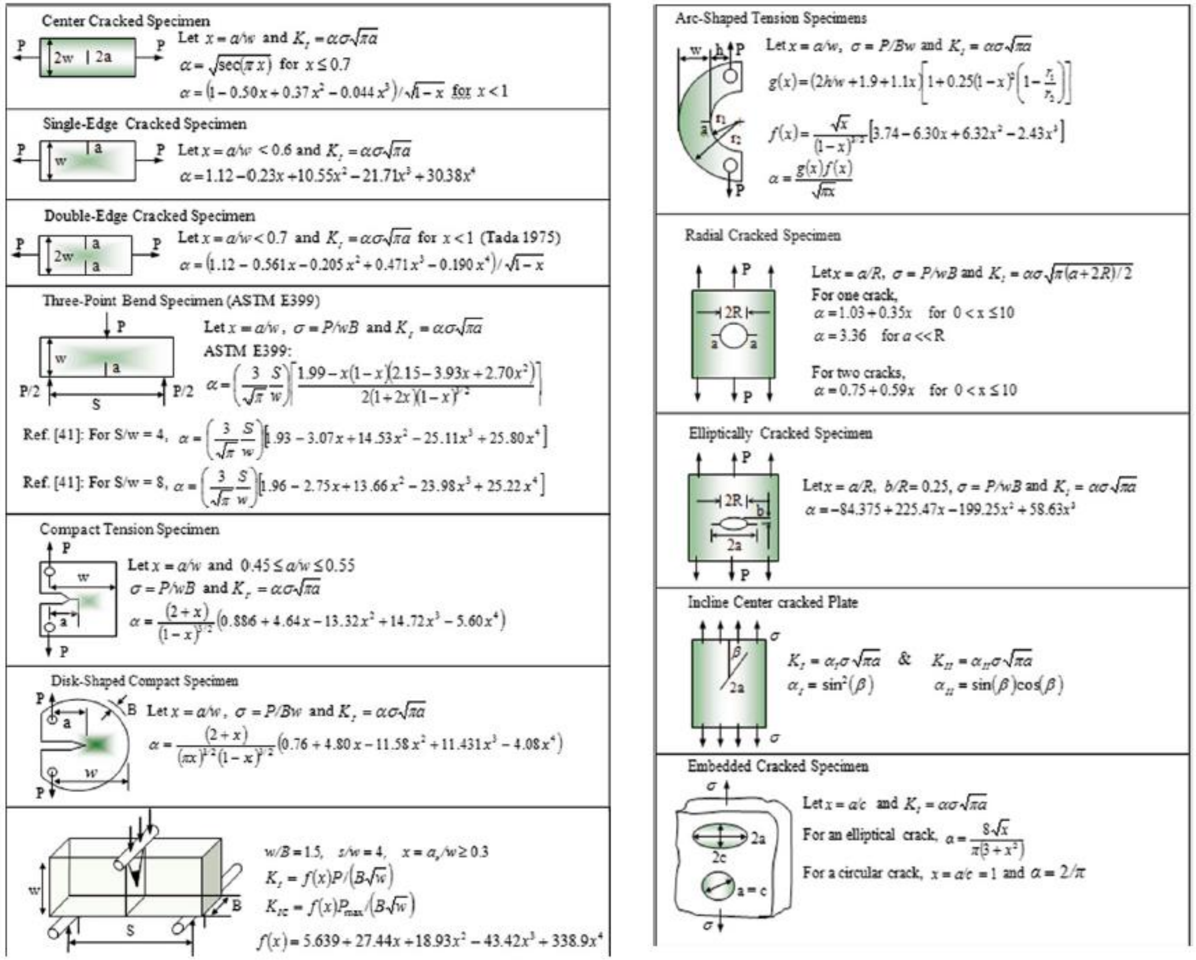


Zone of micromechanical fracture processes at the crack tip



Standards for Computing Stress Intensity Factors

- American Society for Testing and Materials (ASTM)
- European Guidelines SINTAP and FITNETH.



Tada, P.C. Paris, G.R. Irwin, The Stress Analysis of Cracks Handbook, 3rd edn. (ASME Press, New York, 2000)

Energy Balance During Crack Propagation

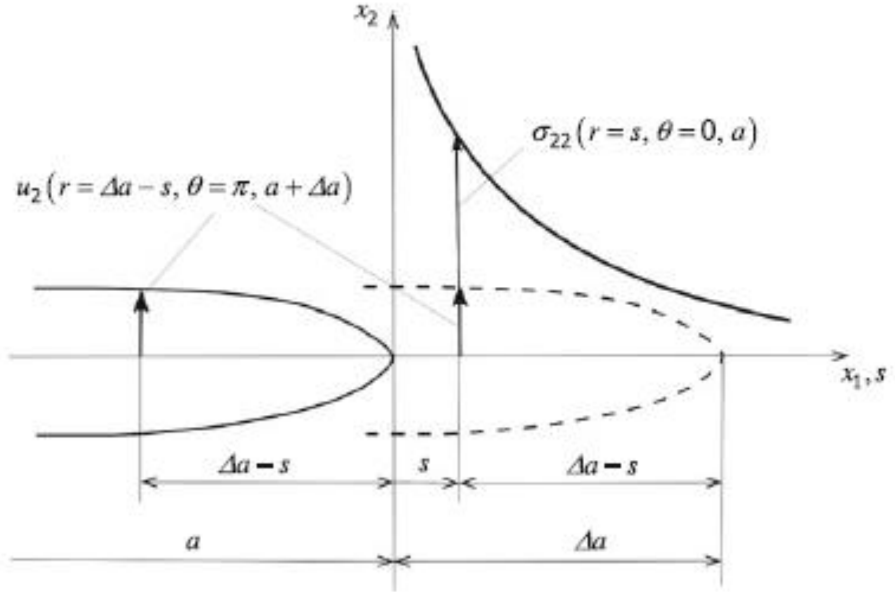
(the 1st law of thermodynamics to a deformable body provides the change in energy per time)

$$\dot{\mathcal{W}}_{\text{ext}} + \dot{\mathcal{Q}} = \dot{\mathcal{W}}_{\text{int}} + \dot{\mathcal{K}} + \dot{\mathcal{D}}\,.$$

- On the left-hand side of the equation stands the energy input into the body per time as
  - the external mechanical loading: $\dot{\mathcal{W}}_{\text{ext}} = \int_{S_t} \bar{t}_i \dot{u}_i \,\mathrm{d}S + \int_V \bar{b}_i \dot{u}_i \,\mathrm{d}V$
  - the exchange of thermal energy $\dot{Q}$ by way of thermal flux or internal heat sources
- On the right-hand side of the balance equation stand those types of energy absorbed by the body per time,
  - the internal energy, which for the purely mechanical: $\mathcal{W}_{\text{int}} = \int_V U \,\mathrm{d}V\,, \quad U(\varepsilon_{kl}) = \int_0^{\varepsilon_{kl}} \sigma_{ij}(\varepsilon_{mn})\,\mathrm{d}\varepsilon_{ij}$
  - the kinetic energy: $\mathcal{K} = \frac{1}{2}\int_V \rho\, \dot{u}_i \dot{u}_i \,\mathrm{d}V$
  - the mainly dissipative energy D, consumed during the crack propagation in the process zone: $\mathcal{D} = 2\gamma A$
- On static problems, so that $K = 0$. For purely elastic deformations $U = U_e$, the internal energy possesses the character of an internal potential $\Pi_{int} = \mathcal{W}_{int}$. Furthermore, we consider the body to be an adiabatically closed system without any internal heat sources, so that $\dot{Q} = 0$ as well.
- Introducing the internal and external potentials, which can be combined to the total potential $\Pi = \Pi_{int} + \Pi_{ext}$, we obtain: $\frac{\Delta(\mathcal{W}_{\text{ext}} - \mathcal{W}_{\text{int}})}{\Delta A} = -\frac{\Delta \Pi}{\Delta A} \stackrel{!}{=} \frac{\Delta \mathcal{D}}{\Delta A} = 2\gamma$
- Hence, **the Energy Release Rate** is defined for finite or infinitesimal crack propagation as follows: $G = -\lim_{\Delta A \to 0} \frac{\Delta \Pi}{\Delta A} = -\frac{\mathrm{d}\Pi}{\mathrm{d}A}$
- Energetic fracture criterion for crack propagation defined by **Griffith**: $-\frac{\mathrm{d}\Pi}{\mathrm{d}A} = G = G_c = 2\gamma$

Link between ERR and SIFs

The Crack Closure Integral: During a virtual load releasing process, the work of the traction on the ligament in front crack performs the work at the crack flank displacements:



$$\Delta W_c = -\int_0^{\Delta a} \frac{1}{2}\sigma_{22}(u_2^+ - u_2^-)\mathrm{d}s = -\int_0^{\Delta a} \frac{K_I(a)}{\sqrt{2\pi s}}\frac{\kappa+1}{2\mu}K_I(a+\Delta a)\sqrt{\frac{\Delta a - s}{2\pi}}\mathrm{d}s$$

$$= -\frac{\kappa+1}{4\pi\mu}K_I(a)K_I(a+\Delta a)\int_0^{\Delta a}\sqrt{\frac{\Delta a - s}{s}}\mathrm{d}s$$

Hence the energy release rate G

$$G = \lim_{\Delta a \to 0}\left(-\frac{\Delta W_c}{B\Delta a}\right) = \frac{\kappa+1}{4\pi\mu}K_I^2(a)\lim_{\Delta a\to 0}\underbrace{\frac{1}{\Delta a}\int_0^{\Delta a}\sqrt{\frac{\Delta a - s}{s}}\mathrm{d}s}_{\pi/2}$$

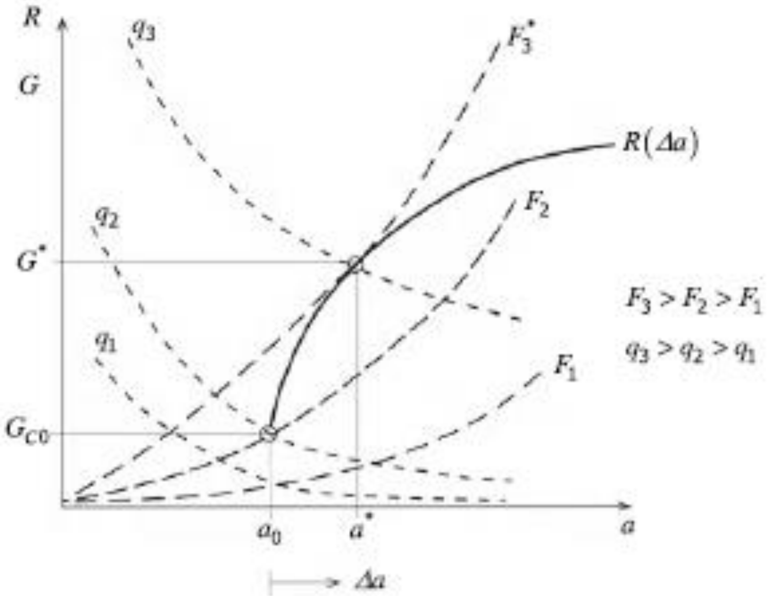
=> $$G_I = \frac{\kappa+1}{8\mu}K_I^2 = K_I^2/E'$$

If mode I, II and III loading are combined, the energy release rate during infinitesimal crack propagation

$$G = G_I + G_{II} + G_{III} = \frac{1}{E'}\left(K_I^2 + K_{II}^2\right) + \frac{1+\nu}{E}K_{III}^2$$

Stability of Crack Propagation

The fracture criterion by Griffith sets the necessary energetic conditions for a crack to be able to propagate at all. In order to assess the further course of crack propagation–especially the issue of stability–it is crucial to see how the fracture condition itself changes.



The material-specific behavior is described by the crack growth resistance curve (R-curve):

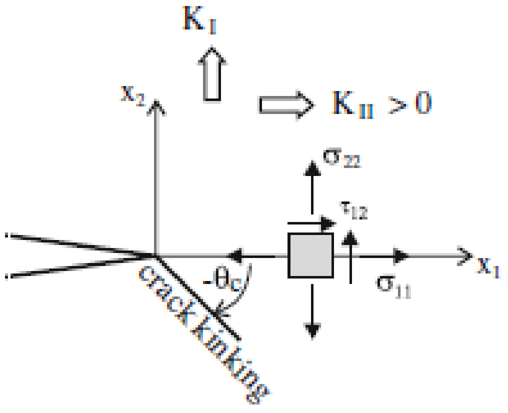
$$G_c = R(\Delta a)$$

Stability of Crack Propagation

$$\left.\frac{\partial G}{\partial a}\right|_{F,q} \begin{matrix} < \\ = \\ > \end{matrix} \frac{\partial R}{\partial a} \quad \begin{cases} \text{stable} \\ \text{indifferent} \\ \text{unstable} \end{cases}$$

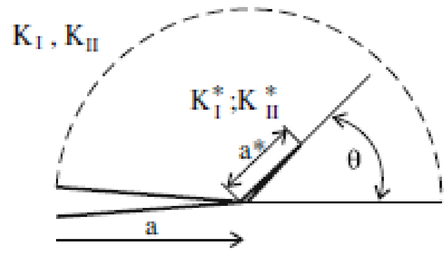
The crack behavior is called *stable*, when the crack resistance *R* increases faster than the driving energy release rate *G*. The crack behavior is regarded as *unstable* as soon as the energy supply increases faster than the crack resistance does.

Fracture Criteria at Mixed-Mode Loading



In engineering practice, there are plenty of examples and reasons leading to mixed-mode loading of cracks:

- structural loads consisting of tension, shear, torsion or complex load cases
- oblique, curved, branched or kinked cracks
- time-varying, dynamic or thermal in service loads
- enforced crack propagation in a direction oblique to the principal stress because of preferred geometrical, material or technological orientations (interfaces, joints, anisotropy).

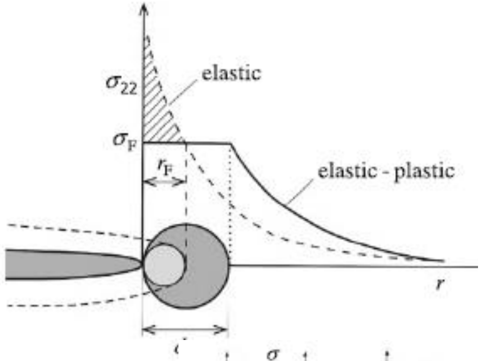


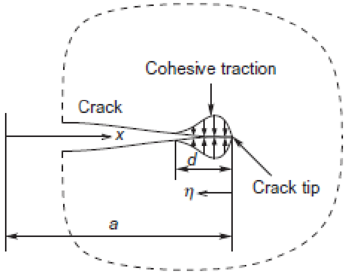
Loading of the crack is defined by the parameter B on the left side, while on the right the critical material parameter B_c characterizes crack initiation. In addition, now a decision has to be made about the direction of crack propagation which deviates from the original direction by an angle θ_c as shown in Fig.

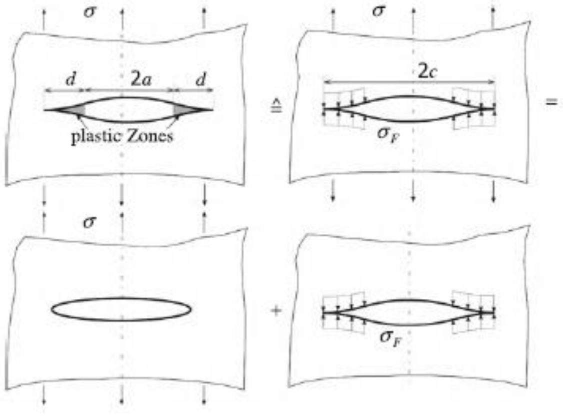
The criterion of maximum circumferential stress was suggested by Erdogan and Sih:
- The crack extends radially from its tip with an angle θ_c in that direction, which is perpendicular to the maximum circumferential stress $\sigma_{\theta\theta max}$.
- Crack propagation initiates, if $\sigma_{\theta\theta max}$ (in a certain distance r_c) reaches a critical material constant σc that corresponds exactly to those under mode I at the ligament if $K_I = K_{Ic}$ is fulfilled.

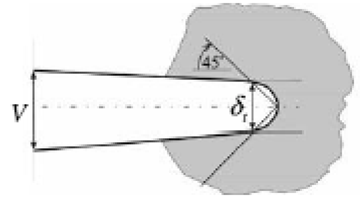
Elasto-Plastic Fracture Mechanics Assumptions

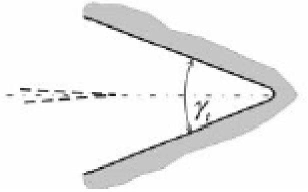
- Small-Scale Yielding (SSY) – *brittle fracture*
  - Irwin's crack length correction, 1957
  - generalisation of Griffith's results to less brittle materials, Orowan, 1945
  - Dugdale's plastic zone, 1960 and Barenblatt's cohesive zone, 1962
  - Elasto-plastic crack tip opening displacement – CTOD-criterion, Wells, 1961 and Burdekin and Stone, 1966; or crack tip opening angle – CTOA-criterion, e.g. Kanninen, 1979
  - J-T two-term stress approximation theory, Rice, 1974
- Large-Scale Yielding (LSY) – *ductile fracture*
  - Slip-line theory (a rigid-perfectly plastic material), McClintock, 1971 and Sähn and Gölder, 1993 and Gross and Seelig, 2001
  - Rice, Rosengren, 1968 and Hutchinson, 1968 (Ramberg-Osgood strain hardening material, i.e. deformation plastic theory) – Rice-Rosengren-Hutchinson (HRR) elastic-plastic fields
  - Elasto-plastic J-integral-based fracture criterion for stationary crack under monotonous loading and a fracture process zone J_R, Begley and Landes, 1972
  - J–Q two-term numerical stress solution, O'Dowd and Shih, 1991
  - J-A_2 three-term asymptotic solution, Yang, 1993 and Yang et al., 1993











Fracture Mechanics of Anisotropic Solids

The elastic material can basically be anisotropic.

- Stroh–Lekhnitski formalism, 1962 and 1988, respectively.
- Sih, Paris and Irwin's asymptotic solution, 1965

The basic conclusion is the radial behavior of the anisotropic crack tip field is characterized by the same – inverse square root singularity of stresses and strains as well as by the square root singularity–dependency of the displacements as in the isotropic–elastic case. The only differences are the angular functions in those equations

Dynamic Fracture Mechanics Assumptions

- Basic Equations of Plane Elastodynamics
- Stationary Cracks under Dynamic Loading: action od external loads is transferred by stress waves through the material to the crack
  - Dynamic stress intensity factor (DSIF), Maue's closed-form solution, 1954
  - Dynamic energy release rate, C. Atkinson and J.D. Eshelby, 1968
  - Dynamic crack initiation toughness $K_{\mathrm{I}}(t) = K_{\mathrm{Id}}(\dot{K}_{\mathrm{I}}, t^{*})$
  - Dynamic fracture criterion (strain rate dependency)
- Fast Running Crack: elastodynamic waves are emitted by highly dynamic rupture processes form the crack itself
  - Yoffe's moving crack solution, 1951
  - Dynamic near crack tip fields at a constant crack propagation speed, Rice, 1968
  - Dynamic elasto-plastic fracture at a constant crack propagation in an ideal plastic material, Slepyn, 1974 and Drugan et al., 1982
  - Dynamic crack growth toughness $K_{\mathrm{I}}(t, \dot{a}) = K_{\mathrm{ID}}(\dot{a}, \dot{K}_{\mathrm{I}}, T)$ or $G_{\mathrm{I}}^{\mathrm{dyn}}(t, \dot{a}) = 2\gamma_{\mathrm{D}}(\dot{a}, \dot{K}_{\mathrm{I}}, T)$

  (Nearly all materials show an enormous increase of fracture toughness K_{ID} with ˙a due to various mechanisms (crack branching, strain rate dependency, adiabatic heating, and so on. Expensive experiments are necessary to determine K_{ID} as a function of crack velocity ˙a)
  - Dynamic crack arrest toughness $K_{\mathrm{I}}(t, \dot{a}) \leq K_{\mathrm{Ia}}$ $K_{\mathrm{Ia}} < K_{\mathrm{ID}}(\dot{a} \to 0) < K_{\mathrm{Id}}$

  (the energy supply falls off, the crack velocity is slowed down and the crack finally stops. It characterizes the ability of a material to absorb a
  running crack.)
  - Subsonic crack propagation speed
  - Supersonic crack propagation speed
- Experiments

Dynamic Fracture Features

- After crack initiation the dynamic cracks almost always proceed in an unstable manner (after a short phase of acceleration, dynamic cracks attain speeds of propagation ranging in the order of sound wave speeds)
- The high strain rates at the crack tip induce in many materials an embrittlement, since viscoplastic effects reduce the energy absorption capacity of the fracture process zone and the fracture toughness decreases
- Due to complicated elastodynamic wave phenomena such as reflection, superposition, dispersion and attenuation, in dynamic fracture processes no longer does a proportional relationship exist between the temporal course of applied load σ(t) and the stress state at the crack $K_I(t)$
- In the beginning the wave phenomena dominate in a short time range, as long as high kinetic energy is in the system. With increasing time this energy is dissipated, and the waves are attenuated, scattered, and finally fade out. The larger the crack is in proportion to the body, the more pronounced is the short-term effect
  - Dynamic overshoot:
  - $K_I(t)$ depends on the ration between an external pulse duration and a Rayleigh wave along the crack length (for long pulses the loading behaves qualitatively similar as in the static case)
- Etc.

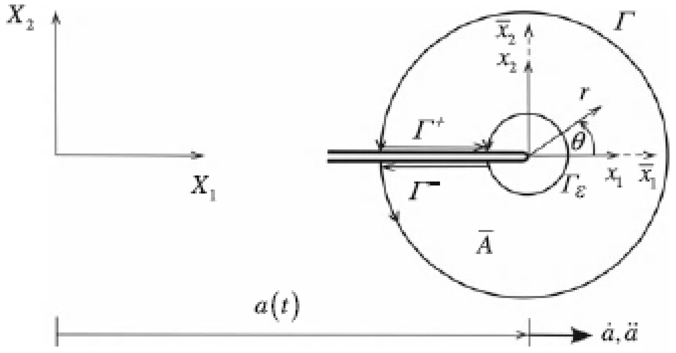
$$\max_t K_{\mathrm{I}}(t) \approx K_{\mathrm{I}}(t_{\max} = 2a/c_{\mathrm{R}}) = \varkappa K_{\mathrm{I}}^{\mathrm{sta}}$$

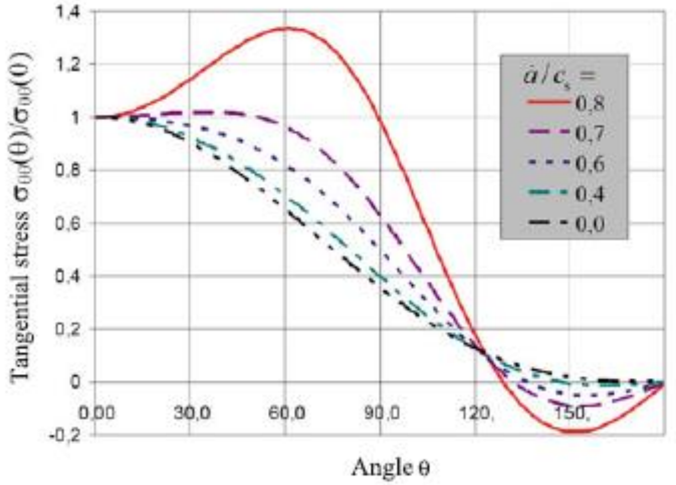
Dynamic Loading of Stationary Cracks

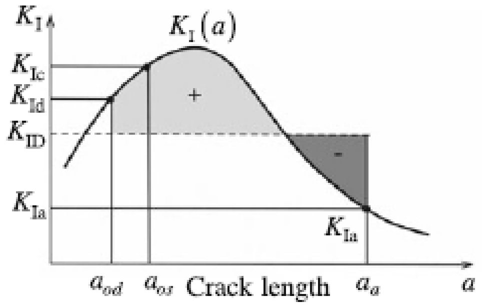
- At the tip of a dynamically loaded, stationary crack exactly the same near-tip fields arise as in the static case. There exist identical singularities and the same separation applies to the three opening modes I, II and III with the stress intensity factors as loading parameters. The only but essential difference to the static case consists in that the K-factors in dynamics depend on time t.
- The calculation of the K-factors as a function of transient load, component geometry, crack length and time requires solution of the elastodynamic IBVP

Dynamic Crack Propagation

- What is the structure of the near-field at the tip of a crack that is propagating with a velocity $\dot{a}$?
- The dynamic crack tip fields have basically the same structure as in statics, but their magnitude and their angular distribution depend on the crack velocity $\dot{a}$, which affects the Rayleigh function and the constants α_d, α_s and γ_d, γ_s.
- the acceleration $\ddot{a}$ of the crack doesn't influence the singular near-field solution but has an impact on higher order terms of the series expansion
- Up to a velocity of $\dot{a} < 0.6\, c_s$ of the shear wave speed the maximum $\sigma_{\theta\theta}$ is located at $\theta = 0$, i.e. in the direction of crack extension. At higher crack velocities the position of the maximum is shifted towards an angle of $\theta=60^\circ$: crack deflection or symmetric crack branching, it can be explained as to relieve the excessive supply of kinetic energy
- A typical scenario of a dynamic fracture process is outlined.







Recommended Literature

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