INTERFACIAL FRACTURE IN SANDWICH COMPOSITES. PART 2: Computational Methods

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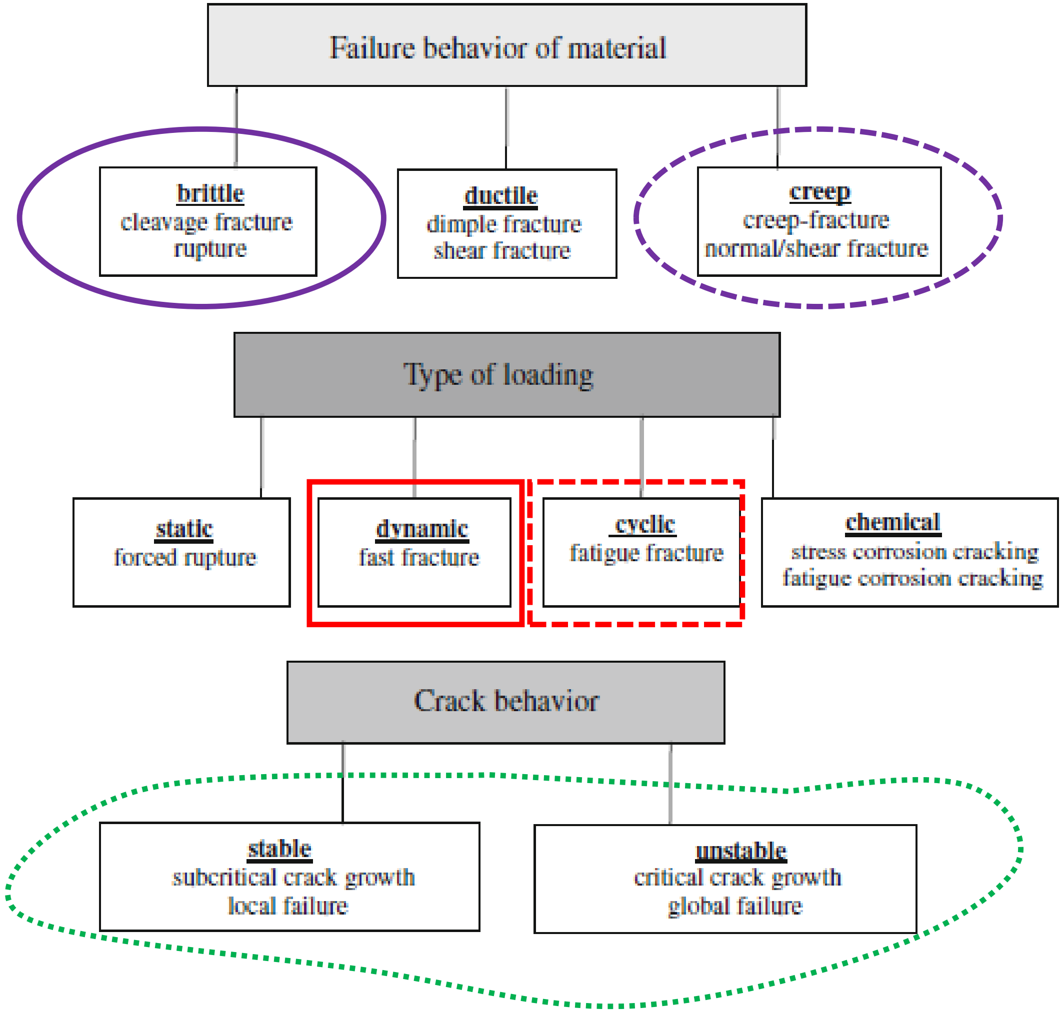
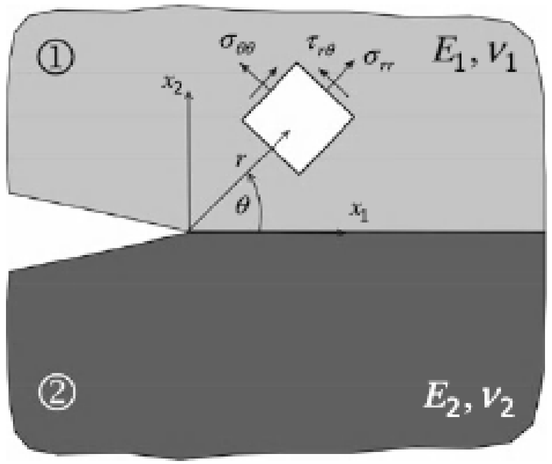
General Classification of Fracture Processes



Interface Fracture Mechanics Assumptions

Often cracks appear in the interface between two materials with different mechanical properties. Such cracks are referred to as interface cracks.



The difference between the elastic properties of both materials is specified by the parameter

$$\epsilon = \frac{1}{2\pi} \ln \frac{\mu_2 \kappa_1 + \mu_1}{\mu_1 \kappa_2 + \mu_2}, \quad 0 \le |\epsilon| \le 0,175$$

Following the Williams solution we arrive at complex stress intensity factor:

$$\widetilde{K} = K_1 + \mathrm{i} K_2 = |\widetilde{K}| e^{\mathrm{i}\psi} / l^{\mathrm{i}\epsilon}, \quad |\widetilde{K}| = \sqrt{K_1^2 + K_2^2}, \quad \psi = \arctan(K_2/K_1)$$

It gives to calculate the stress ahead and the displacement behind crack tip as

$$\sigma_{22}(r, 0) + \mathrm{i}\tau_{12}(r, 0) = \frac{\widetilde{K}}{\sqrt{2\pi r}} r^{\mathrm{i}\epsilon} \quad \Delta u_2(r) + \mathrm{i}\Delta u_1(r) = \frac{8}{1 + 2\mathrm{i}\epsilon} \frac{\widetilde{K}}{E^* \cosh(\pi\epsilon)} \sqrt{\frac{r}{2\pi}} r^{\mathrm{i}\epsilon} \quad r^{\mathrm{i}\epsilon} = e^{\mathrm{i}\epsilon \ln r} = \cos(\epsilon \ln r) + \mathrm{i} \sin(\epsilon \ln r)$$

Generally for interface cracks, $\widetilde{K} = (\sigma_n + \mathrm{i}\tau_n)\sqrt{\pi a}(2a)^{-\mathrm{i}\epsilon} g(a, w, \epsilon)$

Rice (Rice, 1988) suggested a pragmatic way convert this impractical factor to the classical (homogeneous) form:

$$K_{\mathrm{I}} + \mathrm{i} K_{\mathrm{II}} = \widetilde{K} \hat{r}^{\mathrm{i}\epsilon} = (K_1 + \mathrm{i} K_2) \hat{r}^{\mathrm{i}\epsilon} \qquad r_c < \hat{r} \ll a.$$

Alternatively, it is possible to use the energy release rate

$$G = -\lim_{\Delta a \to 0} \frac{\Delta \Pi}{\Delta a} = \frac{K_1^2 + K_2^2}{E^* \cosh^2(\pi\epsilon)}$$

The basic conclusion is: the crack opening modes I and II always occur coupled. Because of the complex product, the crack tip fields cannot be split into separate functions with their own coefficients K_I and K_{II} as in the homogeneous case. That is why the new terms K_1 and K_2 are used instead. Therefore, it is not possible to e. g. relate the crack face displacements u_2 and u_1 or the stresses σ_{22} and τ_{12} ahead of the crack in a unique way to the modes I or II anymore.

Interface Crack Toughness

- the decoupling of the normal and shear components of stress on the interface and associated displacements behind the tip within the zone dominated by the singularity does not occur. Efforts to measure interfacial toughness under mixed mode conditions.
- The interface toughness is not a single material parameter, rather it is a function of the relative amount of mode 2 to mode 1 acting on the interface.
- The criterion for initiation of crack advance in the interface when the crack tip is loaded in mixed mode characterized by ψ is $G=\Gamma(\psi)$. The toughness of the interface $\Gamma(\psi)$ can be thought of as an effective surface energy that depends on the mode of loading
- A Brazil nut specimen enables the experimentalist to vary the mix of loading from pure mode 1 to pure mode 2 by varying the angle of the compression axis, Liechti and Chai, 1990
- Phenomenological mixed mode fracture criteria, e.g. Kinloch, 1987

Theoretical Studies of Interface Crack Problem

- elastic stress and displacement fields at the tip of a open interface crack behave in an oscillatory manner, Williams, 1959 (asymptotic analysis)
- the extent of the oscillatory region is of the order of 10^{-6} of the crack length, Erdogan, 1963
- solutions predict overlapping crack faces near the crack ends – contact area, Cherepanov, 1962, England, 1965 and Rice and Sih, 1965 and Malyshev and Salganik, 1965
- a complex stress intensity factor due to coupling of the stress intensity factors, Hutchinson et al., 1987 and Rice, 1988
- the finite thickness interface models replacing the interface by a thin strip of finite thickness, Atkinson, 1977
- contact zone solution of the Signorini problem at the crack tip, Comninou, 1977
- models based on nonlinear elastic behaviour considerations, Knowles and Sternberg, 1983 or elastic-plastic behaviour, Shih and Asaro, 1988 neglecting the contact area
- methods eliminating the oscillatory portion of singularity, Rice, 1988 and etc.
- experimental studies of a crack between two adherends, e.g. Cao and Evans, 1989 and many others showed that the critical interface energy release rate G_{ic} is a function of the phase angle ψ

Theoretical Methods for Solutions of Singular Stresses in LEFM

- Theory of Complex Functions
  - complex potentials were first introduced by Goursat
  - their applications to problems in elasticity were developed by Kolosov and Muskhelishvili
  - basic techniques used are as follows:
    - conformal mapping
    - Laurent series expansion
    - Wiener-Hopf method
- Integral Transforms
  - most often used transforms are Fourier, Mellin and Hankel transforms
  - as a result the problem often is reduced to an Abel's equation (singular integral equations) and solved directly
- Other
  - Method of Eigenfunction Expansion
  - The Alternating Method
  - Three-dimensional Potential Approach

Finite Element Method for Interfacial Crack SIFs

- Stiffness Derivative FE Technique (or VCE), Park, 1974
- Hybrid Finite Element Technique, Lin and Mar, 1976
- Numerical Crack Flank Displacement method, Smelser, 1979
- Conservation Integrals (mode partitioning), Chen and Shield, 1977
- Virtual Crack Closure Integral, Chow and Atluri, 1995

Numerical Methods for Crack Growth Predictions

- Cohesive zone model (CZM)
  - The idea of the CZM is credited to Barenblatt, 1962
  - Needleman introduced the cohesive element technique in the finite
  - element framework for fracture studies, 1987
- Virtual Crack Closure Technique (VCCT)
  - Virtual crack extension, Park, 1974
  - Modified crack closure, Rybicki and Kanninen, 1977
- Finite Element Method (FEM)
  - Conventional Static and Dynamic Implicit or Explicit Structural Analyses:
    - Remeshing Element Technique
    - Removing Element Technique
    - Embeded Discontinuity into Nodes or Elements
  - Arbitrary Lagrangian Eulerian (ALE)
- Extended Finite Element Method (XFEM)
- Phase Field Method
- Peridynamics Method: Smoothed Particle Hydrodynamic SPH and molecular dynamics MD

Numerical Simulation of Crack Propagation

Fracture Mechanics provides criteria and principles for these cases that specify:
- ✓ at what load level crack propagation starts,
- ✓ in what direction θ_c crack propagation occurs,
- ✓ how large the amount Δa of crack propagation is.

Crack propagation means a change of the BVP, because thereby new boundaries (crack faces) are generated with altered conditions. Consequently, in the FEM analysis we are faced with the problem that the spatial discretization has to be cut and adapted consecutively for the propagating crack.

- **Nodal Release Technique**: a FEM-mesh consists in a disconnection of a node, so that the crack is enlarged by an increment Δa along the element edge up to the next node
  - VCCT
- **Techniques of Element Modification**:
  - *Element Splitting* allows the exact numerical simulation of crack paths by fracture mechanics theory
  - *Element Elimination Technique*, the most highly stressed finite element at the crack tip is removed from the FEM model
  - *Smeared Crack Model*, based on reducing material stiffness values by a damage variable inside the microcrack band
- **Remeshing Techniques**:
  - Moving Crack Tip Elements
  - Adaptive Remeshing Strategies
- **Cohesive Zone Models**:
- **Damage Mechanics Models**

Finite Element Methods in Fracture Mechanics: Stress Intensity Factor

- **Displacement or Stress Interpretation Methods**: they interpret directly available FEM results by simple manual calculations based on the comparisons between the FEM data and the known near filed solutions for the crack tip.
- **Quarter-point Crack Tip Element**: a change in the element the coordinates of nodes to the quarter-point position leads to automatic producing the crack tip field singularities.
- **Hybrid Crack Tip Element** (Embeded Finite Element Method): relies on the hybrid finite element formulation enabling us to embed directly the known asymptotic (or other analytic) near field solutions for the crack tip inside the element.
- **Method of Virtual Crack Extension** (Stiffness Derivative Method):
- **Crack Closure Integral,** Virtual Crack Closure Technique (Modified Crack Closure Integral):
- **FE Computations of Weight Functions**:
- **FE Computations of J-Integral**:
    - Transformation into Equivalent 2-D and 3-D Domain Integrals for Numeric Calculations
    - Interaction Integral Technique for Mode Separation (Partitioning)
    - T-Stresses Calculations

Recommended Literature

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- J. Dundurs, Edge-bonded dissimilar orthogonal elastic wedges under normal and shear loading, J. Appl. Mech. 36 (1969) 650–652.
- G.I. Barenblatt, The mathematical theory of equilibrium cracks in brittle fracture, in: Adv. Appl. Mech. 7 (1962) 55–129.
- A. Needleman, A continuum model for void nucleation by inclusion debonding, J. Appl. Mech. 54 (1987) 525–531.
- E.F. Rybicki, M.F. Kanninen, A finite element calculation of stress intensity factors by a modified crack closure integral, Eng. Fract. Mech. 9 (1977) 931–938.
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