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C ABAQUS User-defined subroutines
C The subroutines implement a 3-D graded element with spatially
C variable mechanical and thermal properties in accordance with
C the rule of mixture into ABAQUS/Standard.
C Also, the subroutines handle for 3-D problems with temperature
C dependent material properties.
C
C If using these codes for research or industrial purposes, please
C cite Vycheslav N. Burlayenko, Tomasz Sadowski & Svetlana
Dimitrova, Three-dimensional free vibration analysis of thermally
C loaded FGM sandwich plates, Materials (2019)
C
C NOTE: FGM plates have a through-the-thickness material
C gradation. Here it means along the Y-axis.
C
      SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
     1 RPL,DDSDDT,DRPLDE,DRPLDT,STRAN,DSTRAN,
     2 TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,MATERL,NDI,NSHR,NTENS,
     3 NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,CELENT,
     4 DFGRD0,DFGRD1,NOEL,NPT,KSLAY,KSPT,KSTEP,KINC)
C
      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 MATERL
      DIMENSION STRESS(NTENS),STATEV(NSTATV),
     1 DDSDDE(NTENS,NTENS),DDSDDT(NTENS),DRPLDE(NTENS),
     2 STRAN(NTENS),DSTRAN(NTENS),TIME(2),PREDEF(1),DPRED(1),
     3 PROPS(NPROPS),COORDS(3),DROT(3,3),
     4 DFGRD0(3,3),DFGRD1(3,3)
C
C     LOCAL ARRAYS
C
      DIMENSION EELAS(6),ETHERM(6),DTHERM(6)
C
      PARAMETER (ZERO=0.D0,ONE=1.D0,TWO=2.D0,TWOTHIRD=2.D0/3.D0,
     1 HALF=1.D0/2.D0, THREE=3.0D0,FOUR=4.0D0,FOURTHIRD=4.D0/3.D0)

      REAL*8 E,NU,MU,LAMBDA,PHAS,TMPR,AEm,AEc,ANUm,ANUc
C
      DATA THICKNESS, VOLC /20.0D0,1.0D0/
C
C     USER SPECIFY PARAMETERS FOR THE SUBROUTINE
C
C     PROPS(1) IS Em : Young’s modulus of metal on bottom surface
C     PROPS(2) IS Ec : Young’s modulus of ceramic on top surface
C     PROPS(3) IS p  : the exponent
C     PROPS(4) IS NUm : Poisson’s coefficient - constant
C     PROPS(5) IS NUc : Poisson’s coefficient – constant
C
      TMPR=TEMP+273
      AEm=PROPS(1)*(1+3.079D-4*TMPR-6.534D-7*TMPR**2)
      AEc=PROPS(2)*(1-3.070D-4*TMPR+2.160D-7*TMPR**2-
     $    8.946D-11*TMPR**3)
      ANUm=PROPS(4)*(1-2.002D-4*TMPR+3.797D-7*TMPR**2)
      ANUc=PROPS(5)
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C
      IF (PROPS(3).EQ.0) THEN
          PHAS = 1
      ELSE IF (PROPS(3).GT.10) THEN
          PHAS = 0
      ELSE
          PHAS = VOLC*(HALF + COORDS(2)/THICKNESS)**PROPS(3)
      END IF
C
C     EFFECTIVE MATERIAL PARAMETERS
C
      E = AEm + (AEc - AEm)*PHAS
      NU = ANUm + (ANUc - ANUm)*PHAS
C
C     DETERMINE THE LAME's CONSTANTS
C
      MU=E/TWO/(ONE + NU)
      LAMBDA=E*NU/(ONE+NU)/(ONE-TWO*NU)
C
C     DETERMINE THE TANGENT(JACOBIAN) MATRIX
C
C     NDI  IS  NUMBER OF NORMAL STRESSES:  SXX, SYY, SZZ
C     NSHR IS  NUMBER OF SHEAR STRESSES:  SXY
C     NTENS  IS  DIMENSION OF CONSTITUTIVE MATRIX,NTENS=NDI + NSHR
C
      DO I=1,NDI
        DO J=1,NDI
           IF (I.EQ.J) THEN
                DDSDDE(I, I)= LAMBDA + TWO*MU
           ELSE
                DDSDDE(I, J)= LAMBDA
           ENDIF
        ENDDO
      ENDDO
C
      DO I=NDI+1,NTENS
              DDSDDE(I, I)= MU
      ENDDO
C
C     DETERMINE THE STRESS AND UPDATE THE STRESS
C
      DO I=1,NTENS
        DO J=1,NTENS
             STRESS(I)=STRESS(I) + DDSDDE(I, J)*DSTRAN(J)
        ENDDO
      ENDDO
C
      RETURN
      END
C-----------------------------------------------------------------
C            UEXPAN SUBROUTINE
C-----------------------------------------------------------------
      SUBROUTINE UEXPAN(EXPAN,DEXPANDT,TEMP,TIME,DTIME,PREDEF,
     $     DPRED,STATEV,CMNAME,NSTATV,NOEL)
C
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      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 CMNAME
      DIMENSION EXPAN(*),DEXPANDT(*),TEMP(2),TIME(2),PREDEF(*),
     $ DPRED(*),STATEV(NSTATV)
      DIMENSION ARRAY(15),JARRAY(15)
      CHARACTER*3 FLGRAY(15)
C
C     USER SPECIFY PARAMETERS FOR THE SUBROUTINE
      PARAMETER (HALF=1.D0/2.D0)
     DATA THICKNESS,VOLC,POWER /20.0D0,1.0D0,10.0D0/
     DATA ALPHAM,ALPHAC /12.33D-006,5.8723D-006/
C
     REAL*8 Y,ALPHA,PHAS,TMPR,ALPHM,ALPHC
C
C     GET COORDINATES OF MATERIAL POINTS USING STATEV(2) OF USDFLD
C       Y = STATEV(2)
C
      TMPR=TEMP(1)+273
      ALPHM=ALPHAM*(1+8.086D-4*TMPR)
      ALPHC=ALPHAC*(1+9.095D-4*TMPR)
C
      IF (KSTEP .EQ. 1 .AND. KINC .EQ. 1) THEN
C     READ COORDINATE
           Y = COORDS(2)
      ELSE
           Y = STATEV(2)
      END IF
C
      IF (POWER.EQ.0.0D0) THEN
          PHAS = 1
      ELSE IF (POWER.GT.10.0D0) THEN
          PHAS = 0
      ELSE
      PHAS = VOLC*(HALF + (Y/THICKNESS))**POWER
      END IF
C
      ALPHA = ALPHM + (ALPHC - ALPHM)*PHAS
C      ALPHA = 0.D0
C
     EXPAN(1) = ALPHA*TEMP(2)
      RETURN
      END
C-----------------------------------------------------------------
C               UMATHT SUBROUTINE
C-----------------------------------------------------------------
      SUBROUTINE UMATHT(U,DUDT,DUDG,FLUX,DFDT,DFDG,STATEV,TEMP,
     $ DTEMP,DTEMDX,TIME,DTIME,PREDEF,DPRED,CMNAME,NTGRD,
     $ NSTATV,PROPS,NPROPS,COORDS,PNEWDT,NOEL,NPT,LAYER,KSPT,
     $ KSTEP,KINC)
C
      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 CMNAME
      DIMENSION DUDG(NTGRD),FLUX(NTGRD),DFDT(NTGRD),
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     $ DFDG(NTGRD,NTGRD),STATEV(NSTATV),DTEMDX(NTGRD),TIME(2),
     $ PREDEF(1),DPRED(1),PROPS(NPROPS),COORDS(3)
C
      PARAMETER (THREE = 3.0D0,HALF=1.D0/2.D0)
      DATA THICKNESS,VOLC,POWER /20.0D0,1.0D0,10.0D0/
C
      REAL*8 Y,COND,SPECHT,PHAS
C
C     KM = PROPS(1)
C     CM = PROPS(2)
C     KC = PROPS(3)
C     CC = PROPS(4)
C     POWER = PROPS(5)
C
      IF (KSTEP .EQ. 1 .AND. KINC .EQ. 1) THEN
C     READ COORDINATE
           Y = COORDS(2)
           STATEV(3) = Y
      ELSE
           Y = STATEV(3)
      END IF
C
      IF (POWER.EQ.0.0D0) THEN
          PHAS = 1
      ELSE IF (POWER.GT.10.0D0) THEN
          PHAS = 0
      ELSE
      PHAS = VOLC*(HALF + (Y/THICKNESS))**POWER
      END IF
C
      COND = PROPS(1)+(PROPS(3) - PROPS(1))*PHAS
      SPECHT=PROPS(2)+(PROPS(4) - PROPS(2))*PHAS
C                     INPUT SPECIFIC HEAT
      DUDT = SPECHT
      DU = DUDT*DTEMP
      U = U + DU
C                     INPUT FLUX = -[K]*{DTEMDX}
      DO I=1, NTGRD
         FLUX(I) = -COND*DTEMDX(I)
      END DO
C                     INPUT ISOTROPIC CONDUCTIVITY
      DO I=1, NTGRD
         DFDG(I,I) = -COND
      END DO
      RETURN
      END
C-----------------------------------------------------------------
C     USDFLD subroutine
C     Assign a variation of the density along the Y-coordinate
C-----------------------------------------------------------------
C
      SUBROUTINE USDFLD(FIELD,STATEV,PNEWDT,DIRECT,T,CELENT,
     1 TIME,DTIME,CMNAME,ORNAME,NFIELD,NSTATV,NOEL,NPT,LAYER,
     2 KSPT,KSTEP,KINC,NDI,NSHR,COORD,JMAC,JMATYP,MATLAYO,
     3 LACCFLA)
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C
      INCLUDE 'aba_param.inc'
C
      CHARACTER*80 CMNAME,ORNAME
      CHARACTER*3  FLGRAY(15)
      DIMENSION FIELD(NFIELD),STATEV(NSTATV),DIRECT(3,3),
     1 T(3,3),TIME(2)
      DIMENSION ARRAY(15),JARRAY(15),JMAC(*),JMATYP(*),
     1 COORD(*)
C
      PARAMETER (HALF=1.D0/2.D0)
      DATA THICKNESS,VOLC,POWER /20.D0,1.0D0,10.0D0/
      DATA RHOm, RHOc /8.166D+003,2.370D+003/
      REAL*8 Y,RHO,PHAS
C
      IF (KSTEP .EQ. 1 .AND. KINC .EQ. 1) THEN
C       READ Y-coordinate
        Y = COORD(2)
      ELSE
        FIELD(1) = STATEV(1)
        Y = STATEV(2)
      END IF
C
      IF (POWER.EQ.0.0D0) THEN
          PHAS = 1
      ELSE IF (POWER.GT.10.0D0) THEN
          PHAS = 0
      ELSE
        PHAS = VOLC*(HALF + (Y/THICKNESS))**POWER
      END IF
C
        RHO = RHOm + (RHOc - RHOm)*PHAS
        FIELD(1) = RHO
C
        STATEV(1) = FIELD(1)
        STATEV(2) = Y
C
      RETURN
      END
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